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BBG-003-001617 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

July – 2021

BSMT - 602 (A) : Mathematics

(Mathematical Analysis - II & Group Theory - II)

(Old Course)

Faculty Code : 003

Subject Code : 001617

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instruction:

1. All questions are **compulsory**
2. Write answer of each question in your main answer sheet.

1. Answer the following questions in briefly 20

(1) Define Field
(2) Find characteristic of the ring $(z, +, \circ)$
(3) Define Constant polynomial
(4) Define commutative ring
(5) Find zero divisor of the ring $(z_6, +_6, \circ_6)$
(6) Define Kernel of a homomorphism
(7) If polynomial $g = (0, 5, -1, 2, 0, 0 \ldots\ldots)$ then find degree of g
(8) Give an example of a ring without unity
(9) Define Division Ring
(10) State the first fundamental theorem of homomorphism
(11) Define : Connected set
(12) What is the greatest lower bound of set $\{ \frac{1}{n} / n \in N \}$
(13) Find $L(e^{3t})$
(14) Determine whether the subset $\{2,5\}$ of metric space R is compact or not
(15) Find $L(e^t\, t)$
(16) Find $L^{-1}\left(\frac{1}{s-1}\right)$
(17) Show that R is not compact set
(18) Define compact set
(19) Determine whether set $\{1,2,3,4,5\}$ is connected or not
(20) Find $L^{-1}\left(\frac{1}{s^2+25}\right)$

2. (a)Attempt any three **6**

(1)Show that the sets A=(1,3) and B=(3,5) are separated sets of metric space R

(2)Show that subset R-{1} is not connected

(3)Show that every finite subset of a metric space is compact

(4) Find Laplace transform of e^{2t} sin3t

(5)Prove that $L[4^{5t}] = \frac{1}{s-5log4}$

(6)Find $L^{-1}\left(\frac{3s+4}{s^2+16}\right)$

(b)Attempt any three **9**

(1) State and prove Bolzano-Weirstrass theorem

(2) Prove that every open interval of metric space R is an open set

(3) If E is a closed subset of metric pace X and H is a compact subset of X Then prove that E∩H is also compact

(4) If L{f(t)} = $\bar{f}$(s) then prove that $L\left[\int_0^t f(u)du\right] = \frac{1}{s}\bar{f}(s)$

(5) Find Inverse Laplace transform of $\log\left(\frac{s+b}{s+a}\right)$

(6) Find Laplace transform of t^2 sin 4t

(c) Attempt any two **10**

(1) If (X, d) is a metric space and E_1 and E_2 are connected subsets of X and $E_1 \cap E_2 \neq \emptyset$ then prove that $E_1 \cup E_2$ is also connected

(2) Prove that continuous image of connected set is connected

(3) State and prove theorem of nested intervals

(4) Prove that $L^{-1}\left(\frac{s}{(s^2+4)^2}\right) = \frac{t}{4}\sin 2t$

(5) Using convolution theorem, find $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$

3. (a) Attempt any three **6**

(1) For element a and b of a ring R, prove that (-a)(-b) = ab

(2) If $\emptyset: (G,*) \to (G',\Delta)$ is Homomorphism. If N is a normal subgroup of G then Prove that $\varphi(a^{-1}) = [\varphi(a)]^{-1}$.

(3) Show that a cyclic group of order eight is homomorphism to a cyclic group of order four

(4) (R,+) and (G,×) are groups. G = {z∈C/ |z|=1} then show that mapping $\emptyset$:R→ G is homomorphism.

(5) f(x)=(1,-2,0,2,0,0....) and g(x)=(1,4,0,0,3,0....)∈ R[x] then find f(x)+g(x).

(6) Let I be an ideal of a ring R with unity. Then prove that I = R if 1∈I

(b) Attempt any three **9**

(1) State and prove Remainder theorem

(2) Let $\emptyset: (G,*) \rightarrow (G', \Delta)$ be a Homomorphism then prove that $k_{\emptyset}$ is a normal subgroup of G

(3) Find all homomorphism's of (Z,+) onto (Z,+).

(4) State and prove factor theorem of polynomials

(5) Give the example which is right ideal but not left ideal.

(6) In R[x], $f(x) = 6x^3 + 5x^2 - 2x + 25$ is divided by $g(x) = 2x^2 - 3x + 5$ then find quotient q(x) and remainder r(x)

(c) Attempt any two **10**

(1) Prove that a Homomorphism $\emptyset: (G,*) \rightarrow (G', \Delta)$ is one-one iff $k_{\emptyset}$ ={e}

(2) State and prove first fundamental theorem of homomorphism.

(3) Find gcd of polynomials $f(x) = X^3 + 3x^2 + 3x + 3$ and $g(x) = 4x^3 + 2x^2 + 2x + 2$ of $Z_5[x]$ and express it of the form a(x)f(x) + b(x)g(x)

(4) State and prove division algorithm for polynomials

(5) Prove that a commutative ring with unity is a field if it has no proper ideal